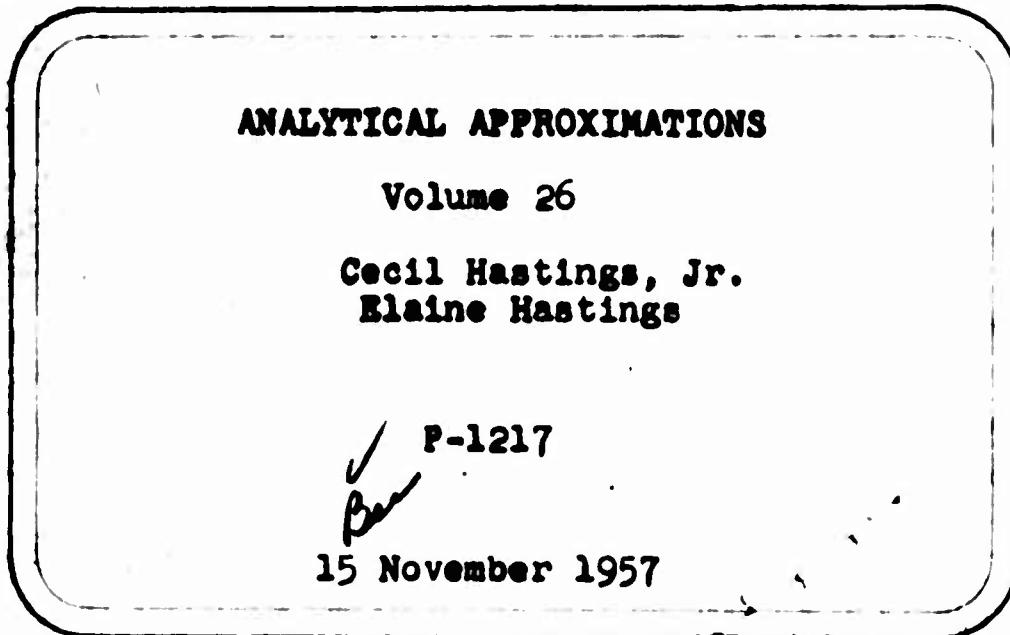


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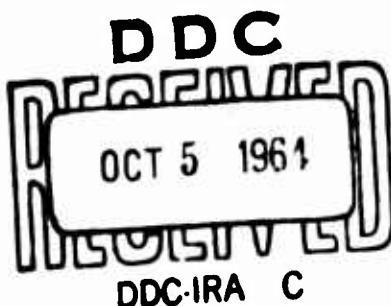
(1)



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### Analytical Approximation

Incomplete Elliptic Integral: To better than  
.0000,0003 over  $45^\circ \leq \phi \leq 90^\circ$  and  $45^\circ \leq \alpha \leq 90^\circ$ ,

$$\begin{aligned} F(\phi, \alpha) &= \int_0^\phi \frac{d\phi}{\sqrt{1-\sin^2 \alpha \sin^2 \phi}} \\ &\approx A(\alpha) + B(\phi, \alpha) \sqrt{1-\sin^2 \alpha \sin^2 \phi} \\ &+ C(\alpha) \ln \left[ \frac{1}{\cos \phi \sin \alpha + \sqrt{1-\sin^2 \alpha \sin^2 \phi}} \right] . \end{aligned}$$

$$\bullet \quad \bullet \quad \bullet$$

$$\begin{aligned} A(\alpha) &= 1.3862,9436 & C(\alpha) &= 1.0 \\ &+.0965,7350(\pi - \alpha)^2 & &+.2499,9980(\pi - \alpha)^2 \\ &-.0013,0437(\pi - \alpha)^4 & &+.0572,9764(\pi - \alpha)^4 \\ &-.0013,7074(\pi - \alpha)^6 & &+.0149,5866(\pi - \alpha)^6 \\ &-.0002,7227(\pi - \alpha)^8 & &+.0047,0849(\pi - \alpha)^8 \\ &-.0000,9495(\pi - \alpha)^{10} & &+.0008,2724(\pi - \alpha)^{10} \\ & & &+.0010,8108(\pi - \alpha)^{12} \end{aligned}$$

Analytical Approximation

$$B(\theta, \alpha) = f_1(\theta)g_1(\alpha) + f_2(\theta)g_2(\alpha) + f_3(\theta)g_3(\alpha)$$

$$\begin{aligned} f_1(\theta) = & -.3277,5088(\frac{1-\theta}{1-\theta})^3 \\ & -.0638,1682(\frac{1-\theta}{1-\theta})^5 \\ & -.0080,8686(\frac{1-\theta}{1-\theta})^7 \\ & -.0008,5024(\frac{1-\theta}{1-\theta})^9 \\ & -.0000,9551(\frac{1-\theta}{1-\theta})^{11} \end{aligned} \quad \begin{aligned} g_1(\alpha) = & .7674,7222 \\ & +.3184,7443(\frac{1-\alpha}{1-\alpha})^2 \\ & +.0820,7991(\frac{1-\alpha}{1-\alpha})^4 \\ & +.0179,8074(\frac{1-\alpha}{1-\alpha})^6 \\ & +.0034,7561(\frac{1-\alpha}{1-\alpha})^8 \\ & +.0013,5976(\frac{1-\alpha}{1-\alpha})^{10} \end{aligned}$$

$$\begin{aligned} f_2(\theta) = & .0298,811 (\frac{1-\theta}{1-\theta})^3 \\ & -.0606,009 (\frac{1-\theta}{1-\theta})^5 \\ & -.0206,017 (\frac{1-\theta}{1-\theta})^7 \\ & -.0040,129 (\frac{1-\theta}{1-\theta})^9 \\ & -.0007,920 (\frac{1-\theta}{1-\theta})^{11} \end{aligned} \quad \begin{aligned} g_2(\alpha) = & .0516,668 \\ & -.1689,896 (\frac{1-\alpha}{1-\alpha})^2 \\ & -.0998,444 (\frac{1-\alpha}{1-\alpha})^4 \\ & -.0344,697 (\frac{1-\alpha}{1-\alpha})^6 \\ & -.0065,684 (\frac{1-\alpha}{1-\alpha})^8 \\ & -.0043,974 (\frac{1-\alpha}{1-\alpha})^{10} \end{aligned}$$

$$\begin{aligned} f_3(\theta) = & -.000727 (\frac{1-\theta}{1-\theta})^3 \\ & +.004444 (\frac{1-\theta}{1-\theta})^5 \\ & -.003986 (\frac{1-\theta}{1-\theta})^7 \\ & -.002520 (\frac{1-\theta}{1-\theta})^9 \\ & -.000621 (\frac{1-\theta}{1-\theta})^{11} \end{aligned} \quad \begin{aligned} g_3(\alpha) = & .005752 \\ & -.075208 (\frac{1-\alpha}{1-\alpha})^2 \\ & +.073873 (\frac{1-\alpha}{1-\alpha})^4 \\ & +.088197 (\frac{1-\alpha}{1-\alpha})^6 \\ & -.014601 (\frac{1-\alpha}{1-\alpha})^8 \\ & +.044161 (\frac{1-\alpha}{1-\alpha})^{10} \end{aligned}$$

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Elaine Hastings  
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